GOOL
בשביל התירגול

מכניקה קלאסית

פרק 11 - הסבר יותר מעמיק על גרדיאנט ורוטור (למי שמעוניין)

תוכן העניינים

אופרטור נאבלה:

רקע:

$$\vec{\nabla} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

	קרטזיות	גליליות	כדוריות
grad $\vec{\nabla} f$	$\frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$	$\frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z}$	$\frac{\partial f}{\partial r}\hat{r} + \frac{1}{r sin\varphi}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r}\frac{\partial f}{\partial \varphi}\hat{z}$
div $\vec{\nabla} \cdot \vec{F}$	$\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$	$\frac{1}{r}\frac{\partial (rF_r)}{\partial r} + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial (r^2 F_r)}{\partial r} + \frac{1}{r \sin\varphi}\frac{\partial F_\theta}{\partial \theta} + \frac{1}{r \sin\varphi}\frac{\partial}{\partial \varphi}(A_\varphi \sin\varphi)$
Rot/curl $\vec{\nabla} \times \vec{F}$	$\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{x} - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right)\hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z}$	$\left(\frac{1}{r}\frac{\partial F_z}{\partial \theta} - \frac{\partial F_\theta}{\partial z}\right)\hat{r} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial (rF_\theta)}{\partial r} - \frac{\partial F_r}{\partial \theta}\right)\hat{z}$	$\frac{1}{r \sin\theta}\left(\frac{\partial}{\partial \theta}(F_\theta \sin\varphi) - \frac{\partial F_\varphi}{\partial \theta}\right)\hat{r} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rF_\varphi) - \frac{\partial F_r}{\partial \varphi}\right)\hat{\theta} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \theta} - \frac{\partial}{\partial r}(r \cdot F_\theta)\right)\hat{\varphi}$

זהויות:

$$\vec{\nabla}(f+g) = \vec{\nabla}f + \vec{\nabla}g$$
$$\vec{\nabla}(\vec{A}+\vec{B}) = (\vec{\nabla}\cdot\vec{A}) + (\vec{\nabla}\cdot\vec{B})$$
$$\vec{\nabla}\times(\vec{A}+\vec{B}) = (\vec{\nabla}\times\vec{A}) + (\vec{\nabla}\times\vec{B})$$
$$\vec{\nabla}(\vec{A}\cdot\vec{B}) = \vec{A}\times(\vec{\nabla}\times\vec{B}) + \vec{B}\times(\vec{\nabla}\times\vec{A}) + (\vec{A}\cdot\vec{\nabla})\vec{B} + (\vec{B}\cdot\vec{\nabla})\vec{A}$$
$$\vec{\nabla}(f\cdot g) = (\vec{\nabla}f)\cdot g + (\vec{\nabla}g)\cdot f$$
$$\vec{\nabla}(f\cdot\vec{A}) = f(\vec{\nabla}\cdot\vec{A}) + \vec{A}\cdot(\vec{\nabla}f)$$